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ELECTRONIC COMPUTERS AND PSYCHOLOGICAL RESEARCH

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ALTHOUGH it is only about ten years since the first high-speed electronic computer was completed, already a number of universities, commercial firms, and governmental agencies have these machines, and many new installations are currently in the planning or the construction stage. Accordingly, it seems not too soon to ask what the effects of electronic computation will be on psychological research.

This paper has the following sections: a brief historical review, a description of the operation of electronic computers, preparation of programs (i.e., machine orders) for computers, advantages and disadvantages of using computers, changes in numerical procedure, the development of new methods, changes in research design. Finally, there is a note of warning about some of the dangers of being able to calculate with less effort. Since my own experience has principally been with Ordvac (Aberdeen Proving Ground) and Illiac (University of Illinois), examples in this paper relate to these two machines. While new programs generally have to be prepared for the different machines because of differences in machine orders, the general principles of electronic computation are sufficiently similar from one installation to another for the Ordvac and Illiac experience to be regarded as typical.

DEVELOPMENT OF ELECTRONIC COMPUTERS

Mathematics and statistics have played an increasingly important part in science during the last 500 years. This quantification of science has not been without disadvantages. The intricacy of the calculations has tended to increase so that the scientist has had to give a larger and larger proportion of his time to the numerical analysis of his results.

Considerably ingenuity has accordingly been displayed in developing computational aids. A fascinating article in the *Encyclopedia Britannica* deals with their development from abacus to slide rule to desk calculator to punched-card machines (20). The latest and most powerful aids are the electronic computers. Their history has been traced back to 1835, when Babbage of the University of Cambridge conceived and designed a mechanical calculating machine, but his device was never completed (5). Interest in the possibility of a very rapid calculating machine revived during the Second World War, and by 1944 the Harvard Mark I computer, a relay machine with moving parts, was ready for use. The first fully electronic machine, the Eniac, built at the University of Pennsylvania for the Army Proving Ground at Aberdeen, Maryland, was completed in 1945. Since then the high-speed computer field has expanded rapidly. In the United States, there must be 100 or more installations in operation at present, and now that their production has been placed on a regular commercial basis, the number is increasing rapidly.

At the same time technical performance has greatly improved. On the Harvard Mark I, a multiplication would take as long as seven seconds, and a division an entire minute. On Eniac, 360 multiplications can be made in a second. Fast modern machines average 1,000 or more multiplications or divisions per second, and even higher speeds are expected in the future.

The first university to make a computer available for general research purposes appears to have been the University of Illinois. The Digital Computer Laboratory was established to provide (*a*) training in the engineering and mathematics basic to electronic computation and (*b*) facilities for extensive calculations by faculty members and graduate students. Two electronic computers have been built on the campus. The first was Ordvac, completed in 1951 for the Aberdeen Proving Ground, and available to university personnel until completion

[1] The work at the University of Illinois reported here has been a cooperative effort involving a number of persons: in particular, K. W. Dickman, J. O. Neuhaus, and R. J. Twery of the Department of Psychology; G. H. Golub and R. T. Gregory of the Digital Computer Laboratory. The work would not have been possible if it had not been for the generous policy of R. E. Meagher, Director of the Digital Computer Laboratory, and of J. P. Nash, Research Professor of Applied Mathematics, of making Illiac freely available to psychologists and social scientists at the University of Illinois.

of the university's own machine, Illiac, the following year. These are two of a group of machines constructed in accordance with a design of Von Neumann, the distinguished mathematician who until his recent death was at the Institute for Advanced Study at Princeton. Computing centers with electronic machines have subsequently been established at various other universities.

OPERATION OF AN ELECTRONIC COMPUTER

For those unfamiliar with computers, a typical Illiac operation will first be described. Data are punched upon teletype tape, a step analogous to preparation of IBM cards. (If data are already recorded on IBM cards, they can be transferred mechanically to tape using the IBM Type 63 card-to-tape machine.) Machine orders for Illiac are punched on another tape. Input of orders and data into Illiac is achieved by running the tapes through a photoelectric reader at the rate of about 200 characters per second.[2] Once input is completed, calculations begin at about 1000 arithmetical operations per second—by an arithmetical operation is meant a completed addition, subtraction, multiplication, or division of binary numbers which are the equivalent of ten-place decimal numbers. As soon as calculations finish, results are punched as decimal numbers on a tape at about 60 characters per second. This completes Illiac use. Results are subsequently printed from the tape by teletype equipment.

TABLE 1

SPEED OF PERFORMANCE ON THE ILLIAC

Arithmetical Operation	Operations per Minute
Addition	650,000
Subtraction	650,000
Multiplication	85,000
Division	75,000
Square Root	2,400
Input and Output of Tape	**Digits per Minute**
Input: Photoelectric Reader	12,000
Output: Teletype (Punched tape)	3,600
Output: Teletype (Printed results)	600
Output: Oscilloscope (Photographic copy)	2,400

Illiac machine speeds are summarized in Table 1. In most current statistical analyses in psychology, computation is so rapid that most of the machine time is needed for input and output of tape. For example, of the 10 minutes or so needed for calculating the intercorrelations of 30 tests for a sample of 500 persons, 90% or more of the time is required for input and output. The correlations are calculated almost instantaneously.

PROGRAMMING FOR AN ELECTRONIC COMPUTER

Just as different methods have been needed for slide rule, desk calculator, and punched-card equipment, so a completely fresh look at calculating techniques is once again required in electronic computation. These changes in computational procedures can be rather well illustrated by considering methods for calculating correlations. (Times have to be regarded as approximations. They vary considerably according to the size of sample, number of digits in the scores, and so on. But even the approximate times given here will indicate the remarkable computational advances of the last 20 years.)

For hand work, squared paper is used for plotting scores. The number of classes is reduced to manageable proportions by grouping scores, although with some loss of accuracy. Some of the additions and multiplications are sufficiently simple to be handled by mental arithmetic, but a slide rule is useful for several lengthier multiplications and divisions and extraction of a square root. After an hour or so, the correlation will be obtained for one pair of tests.

On a desk calculator, the x score for a person is placed on the left-hand side of the register and the y score on the right-hand side. By squaring: x^2, $2xy$, and y^2 are recorded in the upper register, and x and y in the lower. The correlation is calculated from the totals obtained by accumulating these values for successive persons. A correlation can be calculated in about 30 minutes in this way.

On an IBM Tabulator, the operation known as progressive digiting enables products of the x scores to be obtained with a number of other scores simultaneously. If a large set of intercorrelations is needed, 10–20 correlations may be calculated in an hour.

A completely different method has been found the most effective on an electronic computer. All test scores for the first person are input together, and products are calculated and stored. Then the next person's scores are input, and the new products are accumulated with the old. Thus, while conventional methods required us to concentrate upon a

[2] A character refers to a sign, a digit (including certain scaling zeros), or the element of a machine order.

single *test* at a time, forming products for all persons, the electronic computer procedure is to form products for a single *person* for all tests. There is a twofold advantage in this changed order of work: (*a*) any number of persons can be included in the sample despite limited storage capacity, because each person is handled separately; (*b*) test data are generally already recorded in the order required for the electronic computer input. The upshot, as we have seen, is that a correlation matrix for 30 tests —some 435 correlations—can be calculated in about 10 minutes. That is to say, the complete set can be obtained in less time than is needed for a single correlation by the hand method.

Rotation in factor analysis provides another illustration of the changing techniques. While graphical methods are probably as effective as any so long as one is using a desk calculator, they are much less well adapted to an electronic computer, since no precise rule is available for finding the angle of rotation. On a computer, the newer analytic methods of rotation (2, 8, 11, 12, 15, 16, 17) have been found to be much superior.

Desk calculator experience is unfortunately not a reliable guide to the difficulty of preparing a computer program. For example, although the phi coefficient is a simple enough calculation, a computer program proved quite difficult to write. This was because a special system of storage had to be devised utilizing the binary characteristics of the Illiac as fully as possible, thereby enabling many more items to be included in any one analysis.[3] On the other hand, once programs have been written for the basic matrix algebra operations—matrix multiplication, matrix inversion, and the calculation of latent roots and latent vectors—the difficult multivariate calculations, such as canonical analysis (7) and the multiple discriminant function (13), can either be carried out on a step by step basis without any further programming or an integrated program to carry through the entire calculation at once can be prepared relatively easily from the basic programs already available.

Special training is needed for writing programs, and even when a programmer is experienced, several months may be required for preparing a major one. Unfortunately, as much time is sometimes needed for correcting the orders as for the original writing, since errors are rather easily made and can be very difficult to track down. The programmer generally works with limited storage space. For example, the main storage of Illiac consists of only 1024 locations. That is to say, only 1024 numbers and machine orders can be held in the main storage at one time. This is supplemented by 12,800 locations upon a magnetic drum. The programmer tries in planning his program to achieve maximal use of storage space.

Even after programs are in regular use, there are generally many opportunities for improvement: self-checking routines can be written so that the machine checks its own operations at each stage, either stopping or recalculating automatically in the event of any discrepancy; more efficient forms of storage can be developed to enable handling of more data; machine orders can be modified so as to simplify preparation of data and order tapes. The ideal program, in terms of saving labor and machine time, is the integrated one in which a lengthy sequence of calculations is carried out in a single machine run, with such intermediate results printed out as may be required by the investigator and specified to the machine. At present, three separate calculations are required for factor analysis on Illiac. In the first, the correlations are computed. Means, standard deviations, and/or variances and covariances can be obtained in the same operation if required. The second stage is the calculation of principal axes factor loadings (6, 19). The third is the rotation of these loadings by the quartimax or some other analytic method. New teletype tapes are required for each stage at present. The ideal technique will be the combination of these three programs so that the test scores form the input and the output is the rotated factor loadings, along with such intermediate results as are needed by the investigator.

Anyone using an electronic computer should aim at building up a library of basic programs as soon as possible. This is essentially a capital investment. Table 2 lists some of the programs currently available at the University of Illinois. The existing library will be seen to cover many of the standard types of statistical analysis in psychology.

ADVANTAGES AND DISADVANTAGES OF USING COMPUTERS

There are three principal advantages in using electronic computers: rapid computation, less ex-

[3] It might seem at first sight that the general program for product-moment correlation could be used for the calculation of phi coefficients. While this is so, the limited storage space would then be very inefficiently used.

pense, greater accuracy. Little need be said about the first, since the savings in time will be apparent enough from Table 2. This table gives times from the beginning of input of tapes to the end of punching results. Even allowing 10 times or perhaps as much as 20 times as long as this for preparing and

TABLE 2

SELECTED LIST OF PROGRAMS FOR PSYCHOLOGICAL CALCULATIONS ON ILLIAC

Program	Capacity for single machine run	Machine time for capacity run for typical problem (input, calculation, output)	Programmer
A. CORRELATION			
1. Product-moment correlation	38 tests, any number of *Ss*	20 min. for a sample of 400	Twery and Golub
2. Phi coefficients, covariances, and chi squares for dichotomous data	100 items, 300 *Ss*	70 min.	Dickman
3. Intraserial correlations (autocorrelations)	800 3-digit measures	5 min. for lag of 50	Cohn and Augenstine
4. Multiple correlation	21 independent variables	5 min. for a sample of 100 (input of test scores)	Golub
B. MATRIX OPERATIONS			
1. Matrix multiplication $m^A n^B p$	$m + p = 38$, n unlimited	15 min.	Twery
2. Sums and sums of squares for the rows and columns of a matrix	300 rows by 300 columns	5 min.	Neuhaus
3. Matrix inversion	22 tests	5 min.	Frank
4. Calculation of latent roots and vectors	23 tests (latent roots and vectors), 40 tests (latent roots only)	15 min. for order 23	Gregory and Golub
5. Calculation of multiple discriminant function	17 tests	10 min.	Golub
C. FACTOR ANALYSIS			
1. Rao's canonical method	17 tests	Several hours for complete convergence	Golub
2. Principal axes factor analysis	23 tests (40 tests can be handled by a series of operations)	12 min.	Neuhaus
3. Communalities estimation: principal axes analysis	19 tests	4-5 principal axes iterations (i.e., 30 min.)	Twery
4. Centroid factor analysis	60 tests conveniently (more if necessary)	9 factors in 70 min.	Vreenegoor and Dickman
5. Square root factor analysis	250 tests	3-4 factors in 30 min.	Twery
6. Quartimax orthogonal rotation	745 factor loadings	45 min.	Neuhaus
7. Indices of factorial similarity for two or more sets of factors	38 factors	15 min.	Twery
D. GENERAL			
1. Means, standard deviations, variances and covariances	38 tests, any number of *Ss*	15 min. for a sample of 400	Golub
2. Solution of simultaneous equations	39 unknowns	4 min.	Wheeler
3. Linear programming	18 activities, 30 restrictions	15 min.	Isaacson
4. D^2 statistic (Osgood-Cronbach)	30 *Ss*, 30 scores	12 min.	Wilson
5. Intercorrelations of paired comparison data	16 paired elements, 350 *Ss*	12 elements, 100 *Ss*: 20 min.	Stone
6. Roots of a polynomial	300th degree	Roots for 50th degree polynomial in 10 min.	Muller
E. PROGRAMS TO REDUCE CLERICAL LABOR			
1. Matrix transposition	960 elements	5 min.	Twery
2. Ranking of a set of numbers	800 numbers	8 min.	Dickman
3. Generation of random numbers	—	100 3-digit nos. per min.	Augenstine

checking data tapes, printing results by teletype, etc., there are obviously still very great savings in time. It is not exaggerating to say that a job which would take weeks on a desk calculator can be completed in a matter of hours or even minutes with a computer.

The second advantage is cheapness of calculation. This claim may appear incredible since the commercial rental for a high-speed computer may be $200 or more per hour. Indeed, at first sight it seems to be an exceptionally expensive system of computation. But high speed of operation more than offsets high hourly cost. The cost per arithmetical operation on Illiac has been estimated to be only about one-twelfth of that for the university punched-card equipment. Admittedly, it may take a while to convince industrialists, university administrators, etc. that any machine operating at a cost of $100 or more per hour is a bargain. But this is indubitably so. Once this is widely realized, rapid expansion of electronic computation is inevitable. The question to be asked by any large-scale research organization will then no longer be, "Can we afford to use a computer?" but "Can we afford not to use one?"

The third advantage lies in the high accuracy of Illiac. Calculations are generally carried out to ten decimal places, so that final results may be obtained which are accurate to as many as eight decimal places. This means that any results published to three or four places will be completely accurate. Contrast this with any long series of desk calculator operations: the psychologist is often faced with the unpalatable choice of (*a*) working to a large number of decimal places, thereby taking an incommensurately long time in his research, or (*b*) working to very few, when rounding errors may accumulate so much that one can no longer have complete confidence in the final results. Furthermore, there are few desk calculator operators who can calculate day after day without occasional small mistakes, even with a rigorous system of checking. An electronic computer, on the other hand, makes errors much more rarely than a human operator; and either by building the necessary checks into the program, by making an overall check of the final results, or by repeating the analysis on the computer, if need be, complete accuracy becomes attainable.

The principal deterrent to using an electronic computer, assuming one is available, lies in the difficulty of writing programs. The PhD student, for example, has to decide whether the time required for learning to program and for writing programs will be more than saved by the reduced time for computation in his investigation. Likewise, the research administrator may be faced with a similar dilemma as to whether he can afford the time to get the necessary programs prepared or would do better to carry on in his familiar, albeit relatively inefficient, way. These tactical problems will diminish as more extensive libraries of programs become available.

A second disadvantage is likely to be more long enduring. Electronic computation makes greater demands upon staff than do more conventional methods. Indeed, the rate of expansion of electronic computation seems likely to be governed less by supply of machines than by that of trained personnel. The effort in becoming familiar with these machines is quite considerable. Sometimes persons who have successfully enough used a desk calculator in routine clerical fashion prove quite inept at the electronic computer level. In desk calculator work, an occasional error is not overly serious, provided calculations are checked at the end of each stage, since any small section can be recalculated without too much delay. With an electronic computer, any error renders valueless an entire set of results. Furthermore, the exact numerical methods practicable on a computer are generally more demanding mathematically than the ones which they supplant. My impression is that those trained in physical sciences generally make a better transition than those trained in social sciences. The ideal man, of course, is the one with training both in mathematics and the physical sciences and in psychology.

CHANGES IN NUMERICAL PROCEDURE

This paper so far has been concerned with the immediate gain from a computer, viz., the ability to carry out customary analyses more rapidly. Quality of research is unchanged, but the job is done less laboriously.

The greater scientific contribution, however, lies in the widened powers of statistical analysis and experimental design. The remainder of this paper will be concerned with some of these changes. The present section deals with the possibilities of examining data more intensively, of using better statistical methods, and of finding out about the arithmetical consequences of our mathematical assumptions.

The two following sections will be concerned with the development of new mathematical models and with changes in research design.

Our present research designs are often governed more by computational than by logical considerations. Consider, for example, the prediction of some complex performance. The industrial or the clinical psychologist often finds himself with a superabundance of data. But since calculation of a multiple correlation with a large number of predictors is a rather arduous undertaking, the investigator frequently excludes the measures which seem to him least promising prior to his regression analysis. Computational considerations have obliged him to make his selection in advance of his results. A computer changes the position. An initial analysis becomes possible which retains the full set of predictors, and any reduction in their number can then be made on the basis of their regression coefficients.

At present, inferior statistical methods often have to be used even when better ones are available because of the labor of the latter. Prediction of job performance may be given as an example. This performance is often highly complex. A doctor may be a credit to his profession, for instance, because of his great skill in diagnosis, or his clever methods of treatment, or his reassuring bedside manner, or his warm humanity towards his patients, or his achievements in scientific research. He is not necessarily equally successful in all these directions. Yet we are customarily content either to select a single one as our criterion or to sum achievements in these different areas in some arbitrary way. These restrictions upon the criterion are imposed for computational convenience. The mathematical theory exists for handling several criteria simultaneously: Hotelling's canonical analysis was presented 20 years ago and Pearson's bi-multiple correlation 30 years before that. These procedures have remained textbook names because of the excessive calculations. An electronic computer enables us to operate with multiple criteria as well as multiple predictors. A canonical analysis with 20 criteria will not take more than an hour. Once the technique has been used more often, we shall be better able to appraise its usefulness. Some preliminary Illiac results suggest that there may be somewhat similar objections to canonical analysis as those which Thurstone urged against unrotated factor analysis, viz., that the axes in the multivariate system should be located with respect to psychological interpretability rather than mathematical convenience. But even if canonical analysis does not prove to be the final word to the predictive problem, it is certainly logically preferable to our present ad hoc ways of handling multiple criteria (18).

We often do not know much about the arithmetical consequences of our algebra or our statistics. Many examples might be given, but discussion here will be restricted to two. First, although estimates of communalities are extensively used in factor analysis, little is known about the best methods of estimation or the differences introduced in results by varying their values. This is where a computer is helpful. Experiments have shown that estimation is often very inaccurate and that convergence can be extremely slow when the new estimates are derived iteratively from prior loadings. Furthermore, the rank of the matrix can be shown to be much larger than generally assumed (that is, there are many more common factors). More positively, the squared multiple correlation of each variable with the others, already proved by Guttman (4) to be a lower bound for the communality, supplies a reasonably good starting point for the iteration. The second example is drawn from the mathematical theory of learning. Bush and Mosteller (1), considering learning as a stochastic process, have developed a mathematical model for rate of learning. The model is in algebraic terms and does not specify the length of time required by an average animal to learn a given response. The number of trials required has been determined empirically, however, by Twery and Bush by using random responses generated by a computer.

DEVELOPMENT OF NEW METHODS

Electronic computation promises to stimulate the development of more realistic and comprehensive mathematical models because of the knowledge that these can be applied without excessive labor. Three examples can be given from recent research at the University of Illinois.

The first is Rao's canonical method (14) of factor analysis. This supplies a more rigorous statistical formulation for the estimation of factor loadings and the testing of significance of factors, along lines somewhat similar to Lawley's maximum likelihood method (9). Rao developed his procedure with special reference to convenience of electronic computer operation.

The second is Cattell's technique (3) for rotating factors to "parallel proportional profiles." Cattell has proposed that whenever two sets of loadings are obtained for the same variables, the preferred rotations should be those making the loadings of one solution most nearly proportional to those of the other. His recent solution involves algebraic operations convenient upon a computer but laborious by any other means.

The third example is McQuitty's pattern analytic procedures (10). His methods provide a novel attack upon the time-honored problem of attributing different values to item responses depending upon the patterns in which they occur. Illiac programs have brought these patterning techniques within reach of practice even for extensive sets of data.

CHANGES IN RESEARCH DESIGN

Psychological experimentation was originally based upon the physical science model. For the psychophysicist, all factors were to be held constant save the one to be systematically varied to determine its effects. We now know this view of psychological methodology to be too limited to be tenable. First, it is clear that any psychological "laws" will have to be regarded, at least in the foreseeable future, as statements of probability rather than certainty. (A similar shift to probabilistic statements has occurred even in physics.) Secondly, our theory must take account of the patterning and organization of stimuli—in statistical terms, interactions have to be considered as well as main effects. This tendency for psychology to become both probabilistic and multivariate implies increasing recourse to rather complex methods of analysis. Computers will provide the technical assistance needed for this type of research.

The applied psychologist also stands to benefit. At present, in planning any multivariate investigation—a predictive study or a factor analysis—the number of *Ss* to be tested is customarily arranged at the outset of the investigation. This decision usually represents no more than some a priori conviction. Logically a sequential approach would be better. Once a multiple correlation or a factor analysis can be completed more quickly, it becomes practicable to make an initial analysis for the first 100 *Ss*, let us say. If this analysis indicates deficiencies either in the selection or the construction of tests, the testing program can be revised without too great loss of effort. If the battery seems satisfactory, successive analyses can be made for 200 *Ss*, then 300, and so on, until stability is attained in the results. Computers will thereby make possible a more flexible approach to the testing situation, with the possibility of continuing experimentation for as long as may be needed to provide definitive results.

In summary, computers are probably going to have a marked effect upon the planning as well as the analysis of experiments. The reduced proportion of time devoted to statistical analysis will mean that the design can be governed more by logical and less by computational considerations, that statistical assumptions can be examined instead of taken upon trust, and that methods with adequate mathematical rationale can be adopted in place of ad hoc procedures.

A NOTE OF WARNING

There are, of course, dangers in easier computing. For one thing, it will be very easy to collect more statistical results than one can possibly study or report. This may have happened sometimes with desk calculators and punched-card equipment; it will probably happen more often when electronic computers are the all-too-willing slaves. Calculating may become so easy that numbers may sometimes be collected for their own sake. Probably every person working in quantitative psychology has had inquiries along the following lines: "I have 300 variables. Can you tell me what I should do with them?" or, "I should like to examine every possible combination . . ." Furthermore, the psychologist who punches his tapes and leaves Illiac to do the rest will probably have a very imperfect feel for his data as compared with the one who spends days or even weeks calculating his results.

One can only hope that mathematical knowledge will increase as arithmetical skill declines. It is clear that electronic computers will bring psychologists into greater contact with physicists and applied mathematicians. Perhaps some curb on numerical overexuberance will be provided by the consideration that electronic computer time is usually rationed rather carefully and is generally under the watchful control of physical scientists who are not always impressed by the present level of quantitative sophistication in psychology and the social sciences.

For the psychologist skilled in investigating quantitative problems, the electronic computer will be a godsend. His computational load will be considerably lightened, exact methods can be used where approximations have had to suffice, less effort need be given to the development of short cuts, and more time will be available for his principal work, the formulation and testing of hypotheses.

There probably will be excesses and wasted effort in the early days of widespread electronic computer work. Yet, in the long term, it seems to me that electronic computers are going to play much the same rôle for the statistically minded psychologist as the telescope has for the astronomer and the microscope for the biologist: the role of an instrument leading to greatly widened horizons.

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